ANSWERS

TO THE

PROBLEMS IN THE ELEMENTS

OF

PLANE AND SPHERICAL TRIGONOMETRY

By

DAVID A. ROTHROCK

THE MACMILLAN COMPANY
NEW YORK MDCCCCXXI

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ROTHROCK'S TRIGONOMETRY

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TO EXERCISES IN PLANE TRIGONOMETRY

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21. $a = 16$. **22.** $a = 14.4$. **23.** $a = 12$. **24.** $b = 9$. **25.** $b = 16\frac{2}{3}$. **26.** $c = 24$. **27.** $a = 7\sqrt{5}$, $b = 14$. **28.** $a = 100\sqrt{6}$, $c = 250$. **29.** Area $= 200$. **30.** Area $= 2500\sqrt{21} = 11{,}456.4$.

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21. 68. **22.** $\frac{1}{5}\sqrt{11}$. **23.** 0.3. **24.** $48\sqrt{6}$. **25.** $(\frac{8}{5})^3$. **26.** $(\frac{12}{13})^2$. **27.** 1. **28.** 0. **29.** ∞. **30.** $\frac{3}{2}\sqrt{2}$.

31. $\sin\alpha(1-\sin^2\alpha) - \dfrac{1}{\sin^2\alpha} + \dfrac{\sin\alpha}{1-\sin^2\alpha}$. **32.** $1 + \sin\alpha - \dfrac{\sin\alpha}{1-\sin^2\alpha}$. **33.** $1 + \left(\dfrac{\sin\alpha}{\sqrt{1-\sin^2\alpha}}\right)^3 - \dfrac{1}{\sin^2\alpha} + 2\sin\alpha\sqrt{1-\sin^2\alpha}$.

34. $\dfrac{1}{(1-\sin^2\alpha)^2} - \dfrac{\sin^2\alpha}{1-\sin^2\alpha} - \dfrac{(1-\sin^2\alpha)^2}{\sin^2\alpha}$.

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35. $\dfrac{1}{\tan\alpha} + \sqrt{1+\tan^2\alpha} - \dfrac{\sqrt{1+\tan^2\alpha}}{\tan\alpha}$. **36.** $\dfrac{\tan^2\alpha}{1+\tan^2\alpha} + \dfrac{1-\tan^2\alpha}{\tan\alpha}$. **37.** $\dfrac{2+2\tan\alpha}{\sqrt{1+\tan^2\alpha}}$. **38.** $\dfrac{1-\tan\alpha+\tan^2\alpha}{1+\tan\alpha+\tan^2\alpha} - \dfrac{1}{\sqrt{1+\tan^2\alpha}}$. **39.** $\dfrac{(1+\cos\alpha)(1-\cos^2\alpha)}{\cos^2\alpha}$. **40.** $\dfrac{1}{\cos\alpha}$.

41. $\dfrac{\cos^4\alpha + (1-\cos^2\alpha)^2}{\cos^2\alpha(1-\cos^2\alpha)}$. **42.** $\cos\alpha\sqrt{1-\cos^2\alpha}$. **44.** $\cos x = \sqrt{1-\sin^2 x} = \dfrac{1}{\sqrt{1+\tan^2 x}} = \dfrac{\cot x}{\sqrt{1+\cot^2 x}} = \dfrac{1}{\sec x} = \dfrac{\sqrt{\csc^2 x - 1}}{\csc x}$.

45. $\tan x = \frac{\sin x}{\sqrt{1-\sin^2 x}} = \frac{\sqrt{1-\cos^2 x}}{\cos x} = \frac{1}{\cot x} = \sqrt{\sec^2 x - 1} = \frac{1}{\sqrt{\csc^2 x - 1}}$. **46.** $\cos x = \sqrt{1-\sin^2 x}$, $\tan x = \frac{\sin x}{\sqrt{1-\sin^2 x}}$, $\cot x = \frac{\sqrt{1-\sin^2 x}}{\sin x}$, $\sec x = \frac{1}{\sqrt{1-\sin^2 x}}$, $\csc x = \frac{1}{\sin x}$.

47. (1) $\sin x = \sqrt{1-\cos^2 x}$, $\tan x = \frac{\sqrt{1-\cos^2 x}}{\cos x}$, etc.; (2) $\sin x = \frac{\tan x}{\sqrt{1+\tan^2 x}}$, $\cos x = \frac{1}{\sqrt{1+\tan^2 x}}$, etc.; (3) $\sin x = \frac{1}{\sqrt{1+\cot^2 x}}$, $\cos x = \frac{\cot x}{\sqrt{1+\cot^2 x}}$, etc.

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1. $200 \cot 35° = 285.6$. **2.** 875.66 (by use of logarithms). **3.** $5000 \cot 30° = 5000\sqrt{3} = 8660$. **4.** $\frac{1}{2}(42)^2 \cot 67° = 373.968$. **5.** $\frac{1}{4}(1225) \sin 45° = 216.54$. **6.** 1125. **7.** 733.01 (by use of logarithms). **8.** 4050. **9.** 1248. **10.** 880.2.

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3. (1) $\frac{3}{4}$; (2) 0; (3) $\frac{5}{2}\sqrt{2}$; (4) $\frac{2}{3}\sqrt{3}\cot 50°$; (5) 1; (6) 0; (7) 1; (8) 0; (9) 2; (10) 1.

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7. 0. **8.** 1. **9.** (1) +; (2) −; (3) 0; (4) −; (5) −. **10.** (1) 135°, 315°; (2) 60°, 120°; (3) 120°, 240°.

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21. $-\sin 20°$. **22.** $\tan 0°$. **23.** $\cos 90°$. **24.** $\cot 20°$. **25.** $-\sin 80°$. **26.** $-\cot 60°$. **27.** $-\sec 20°$. **28.** $\tan 80°$. **29.** $-\sin 40°$. **30.** $-\cos 50°$. **31.** $-\sin 40° \tan 7°$.

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1. 90°, 60°, 45°, 30°, 120°, 135°. **2.** $\frac{\pi}{10}, \frac{31}{45}\pi, \frac{29}{18}\pi, \frac{\pi}{6}, \frac{100.15}{36}\pi$. **3.** 85° 56′ 37.2″, 21.486°, 94 5379°, 157.563°, −22.918°, 305.577°.

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4. (*a*) 79.7957°; (*b*) 27.2957°; (*c*) 54.0563°; (*d*) 390°; (*e*) 22°; (*f*) 51.8873°. **7.** (*a*) 80°; (*b*) 300°; (*c*) 18°; (*d*) 160°; (*e*) 60°; (*f*) 140°; (*g*) 120°.

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15. $\sin(x-y)=\frac{\sqrt{2}}{10}$, $\tan(x+y)=-7$. **16.** $\tan(x-y)=0$, $\cos(x-y)=1$. **17.** $\sin(x+y)=\frac{1}{2}\sqrt{2}$, $\cot(x+y)=1$.

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1. $\sin 2x=\frac{24}{25}$, $\tan 2x=\frac{24}{7}$. **2.** $\tan 2x=-\frac{120}{119}$, $\cos 2x=-\frac{119}{169}$. **3.** $\sin 2x=\frac{1}{2}\sqrt{3}$, $\cos 2x=\frac{1}{2}$. **4.** $\frac{3}{50}\sqrt{91}$. **5.** $\frac{120}{119}$. **6.** $\frac{1}{4}(\sqrt{6}-\sqrt{2})$. **7.** $\frac{2}{3}$. **8.** $\sqrt{3}$. **9.** $\frac{4}{3}$. **10.** $-\frac{1}{9}$. **11.** $\frac{118}{125}$. **12.** $\frac{299}{970}$. **13.** $\sin 22° 30' = \frac{1}{2}\sqrt{2-\sqrt{2}}$, $\cos 22° 30' = \frac{1}{2}\sqrt{2+\sqrt{2}}$. **14.** $2-\sqrt{3}$. **15.** $\frac{1}{2}\sqrt{2-\sqrt{2}}(1+\sqrt{2})$.

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1. 2 sin 25° cos 15°. **2.** 2 sin 55° cos 25°. **3.** 2 cos 50° cos 10°. **4.** 2 cos 55° sin 15°. **5.** 2 cos 35° cos 7°. **6.** 2 sin 35° sin 7°. **7.** 2 cos 40° cos 15°. **8.** 2 sin 15° cos 25°. **9.** −2 sin 50° sin 30°. **10.** $2\cos\frac{\pi}{4}\cos\frac{\pi}{12}$. **11.** $2\sin\frac{7}{2}\theta\cos\frac{\theta}{2}$. **12.** $2\cos 2\theta\sin\theta$.

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13. $-2\sin n\theta\sin\theta$. **14.** $2\cos\frac{n\pi}{4}\sin\frac{\pi}{4}$. **15.** sin 60° + sin 20°.

16. $\sin 90° - \sin 10°$. **17.** $\cos 20° - \cos 60°$. **18.** $\cos 70° + \cos 50°$.
19. $\sin \pi - \sin \frac{\pi}{2}$. **20.** $\sin (2n - 1)\theta + \sin \theta$.

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1. $c = 153.95$, $b = 182.46$. **2.** $c = 4007.8$, $a = 5923$. **3.** $B = 50° 4'$, $C = 60° 35' 50''$, $c = 138.36$. **4.** $A_1 = 55° 17' 33''$, $C_1 = 89° 59' 58''$, $c_1 = 2811.187$, $A_2 = 124° 42' 27''$, $C_2 = 20° 38' 4''$, $c_2 = 988.375$. **5.** $B = 5° 56' 39''$, $C = 112° 39' 3''$, $c = 2003.3$. **6.** Impossible. **7.** $a = 7061.6$, $c = 7432.8$.

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1. $A = 115° 59' 21.5''$, $C = 29° 18' 9.5''$, $b = 178.02$. **2.** $B = 67° 48' 15''$, $C = 33° 50' 5''$, $a = 313.12$. **3.** $B = 11° 10' 29''$, $C = 38° 40' 7''$, $a = 786.25$. **4.** $B = 131° 45' 41''$, $A = 0° 26' 7''$, $c = 9895.0$ (5-place Tables), or $B = 131° 45' 37.3''$, $A = 0° 26' 10.7''$, $c = 9871.88$ (using Bremeker's 6-place Tables). **5.** $B = 17° 27' 25.6''$, $C = 142° 58' 10.4''$, $a = 1267.14$. **6.** $A = 7° 31' 40''$, $B = 63° 8' 44''$, $c = 10330.7$. **7.** $B = 25° 34' 57.8''$, $C = 92° 10' 22.2''$, $a = 1.1025$. **8.** $B = 55° 2' 56''$, $C = 40° 21' 4''$, $a = 285.07$. **9.** $A = 19° 6' 48.6''$, $B = 36° 43' 11.4''$, $c = 4793.3$. **10.** $A = 28° 45' 22''$, $C = 74° 54' 14''$, $b = 9.9396$.

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1. $a = 10.81$. **2.** $b = 13.49$. **3.** $A = 33° 33' 30''$. **4.** $C = 133° 26'$. **5.** $a = 382.04$. **6.** $c = 247.95$. **7.** Perimeter $= 687.1$. **8.** Area $= 1568.75$.

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Using 6-place tables the answers are as follows:

1. $A = 44° 29' 48.4''$, $B = 48° 9' 14.0''$, $C = 87° 20' 57.6''$.
2. $A = 83° 20' 11.8''$, $B = 58° 39' 28.8''$, $C = 38° 0' 19.0''$.
3. $A = 29° 34' 29.6''$, $B = 57° 40' 5.6''$, $C = 92° 45' 24.8''$.

4. $A = 33° 0' 35.0''$, $B = 64° 59' 40.0''$, $C = 81° 59' 44.8''$.
5. $A = 26° 33' 48.8'$, $B = 32° 18' 27.3''$, $C = 121° 7' 43.9''$.

(The above answers will be slightly modified when calculated by use of 5-place tables.)

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1. $C = 55° 20'$, $b = 5676.87$, $c = 6639.8$. **2.** $C = 127° 30'$, $b = 132.09$, $c = 192.33$. **3.** $A = 43° 35' 52''$, $B = 81° 19' 8''$, $c = 74.496$. **4.** $A = 67° 28' 51''$, $B = 57° 19' 5''$, $c = 1840.9$. **5.** $A = 75° 26' 33.3''$, $B = 32° 18' 7.7''$, $c = 0.20015$. **6.** $A = 12° 13' 33.5''$, $C = 146° 15' 26.5''$, $c = 127.21$. **7.** $A = 67° 40' 14.4''$, $C = 44° 9' 21.6''$, $c = 631.50$. **8.** $A = 19° 19' 3\frac{1}{3}''$, $C = 142° 59' 47\frac{2}{3}''$, $c = 89.15$, or $A_1 = 160° 40' 56\frac{2}{3}''$, $C_1 = 1° 37' 54\frac{1}{3}''$, $c_1 = 4.2179$. **9.** Impossible. **10.** $B = 33° 57' 42''$, $C = 82° 45'$, $b = 589.9$, or $B_1 = 19° 27' 42''$, $C_1 = 97° 15'$, $b_1 = 373.14$. **11.** $A = 38° 12' 48''$, $B = 60'$, $C = 81° 47' 12''$. **12.** $A = 106° 46' 42''$, $B = 46° 53' 12''$, $C = 26° 20' 7''$. **13.** $A = 41° 24' 35''$, $B = 55° 46' 16''$, $C = 82° 49' 9''$. **14.** $A = 12° 54' 31.4''$, $B = 147° 9' 31.2''$, $C = 19° 55' 56.2''$. **15.** $A = 50° 29' 15''$, $B = 50° 54' 32''$, $C = 78° 36' 19''$. **16.** $K = 4{,}704.5$, (use logarithms in Ex. 16–24). **17.** $K = 89{,}318.0$. **18.** $K = 131{,}042$. **19.** $K = 86{,}730.6$. **20.** $K = 1{,}885.0$. **21.** $K = 86{,}342$. **22.** $K = 857.48$. **23.** $K = 11{,}876.7$. **24.** $K = 7{,}269.4$.

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1. $\frac{1}{2}\sqrt{3}$. **2.** $\frac{3}{4}$. **3.** $\frac{5}{17}$. **4.** $\frac{1}{3}\sqrt{3}$. **5.** $\frac{1}{2}\sqrt{2}$. **6.** 0. **7.** 1. **8.** 0. **9.** $\sqrt{1 - a^2}$. **10.** 0. **11.** $\frac{1}{2}\sqrt{3}$. **12.** 0. **13.** 1. **14.** $\frac{15}{17}$.

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10. 0°, ± 90°, ± 180°.

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In the following answers, r is taken positive and the smallest trigonometric angles are indicated.

2. $r = 13,\ \phi = 22^\circ\ 37'\ 12''$. **3.** $r = \sqrt{180},\ \phi = 63^\circ\ 26'\ 6''$. **4.** $r = 10,\ \phi = 30^\circ$. **5.** $r = 5.427,\ \phi = 63^\circ\ 49'\ 50.6''$.

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1. $r = 3,\ \phi = 45^\circ,\ \theta = 72^\circ\ 32'\ 40''$. **2.** $r = 7,\ \phi = 26^\circ\ 34',\ \theta = 73^\circ\ 24'$. **3.** $r = 11,\ \phi = 71^\circ\ 33'\ 54'',\ \theta = 35^\circ\ 5'\ 47''$. **4.** $r = 17,\ \phi = 45^\circ,\ \theta = 86^\circ\ 37'\ 37''$. **5.** $r = 9,\ \phi = 75^\circ\ 57'\ 49'',\ \theta = 27^\circ\ 17'$. **6.** $r = \sqrt{29},\ \phi = 21^\circ\ 48',\ \theta = 90^\circ$.

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2. $x = 12^\circ\ 21'\ 6''$. **3.** $x = 139^\circ\ 55'\ 28''$ or $-(5^\circ\ 9'\ 52'')$. **4.** $x = 134^\circ\ 28'\ 19''$ or $-(10^\circ\ 37'\ 1'')$. **5.** $x = 19^\circ\ 26'\ 36''$ or $-(86^\circ\ 49'\ 24'')$. **6.** $x = 72^\circ\ 13'\ 13''$ or $208^\circ\ 10'\ 13''$.

In each answer above multiple angles may be used giving any number of additional answers.

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2. $x = 19^\circ\ 19'\ 18.5''$. **3.** $x = 66^\circ\ 16'\ 55.4''$. **4.** $x = 21^\circ\ 36'\ 55.2''$. **5.** $x = 13^\circ\ 26'\ 25.1''$. **6.** $x = 147^\circ\ 39'\ 22.6''$.

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2. $x = 7^\circ\ 12'\ 48.5''$. **3.** $x = 15^\circ\ 39'\ 15''$. **4.** $x = 3^\circ\ 29'\ 27''$. **5.** Solution impossible.

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1. $x = 45^\circ\ 20'\ 20''$. **2.** $x = 91^\circ\ 59'$. **3.** $x = 79^\circ\ 6'$. **4.** $x = 30^\circ\ 50'$. **5.** $x = 62^\circ\ 23'$.

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2. (1) $r = \sqrt{13},\ \phi = \tan^{-1}\left(\frac{2}{3}\right)$; (2) $r = \sqrt{40},\ \phi = \tan^{-1}(3)$; (3) $r = 3\sqrt{2},\ \phi = \tan^{-1}(-1)$; (4) $r = \sqrt{17},\ \phi = \tan^{-1}\left(-\frac{1}{4}\right)$; (5) $r = 5,\ \phi = \tan^{-1}\frac{4}{3} + \pi$; (6) $r = 6,\ \phi = -90^\circ$; (7) $r = \sqrt{41},\ \phi = \tan^{-1}\left(\frac{-5}{4}\right)$; (8) $r = 5,\ \phi = 60^\circ$. **3.** (1) $z = 2 \pm 3\sqrt{-1} = 2 \pm 3\,i$; (2) $z = -3 \pm 2\,i$; (3) $z = \frac{-1 \pm \sqrt{3}\,i}{2} = \omega,\ \omega^2$; (4) $z = 1,\ \omega,\ \omega^2$; (5) $z = -1,\ -\omega,\ -\omega^2$.

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2. (1) $z = \pm i = \cos 90^\circ \pm i \sin 90^\circ$; (2) $z = 1 \pm i = \sqrt{2}(\cos 45^\circ \pm i \sin 45^\circ)$; (3) $z = \frac{1}{2} \pm \frac{1}{2} i = \frac{1}{\sqrt{2}}(\cos 45^\circ \pm i \sin 45^\circ)$; (4) $z = 2$, $-4 = 2(\cos \theta + i \sin \theta)$, $4(\cos 180^\circ + i \sin 180^\circ)$; (5) $z = \frac{-1 \pm i}{\sqrt{2}} = \cos 135^\circ \pm i \sin 135^\circ$; (6) $z = \frac{-1 \pm \sqrt{3}\, i}{2} = \cos 120^\circ \pm i \sin 120^\circ$.

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2. $z = \cos 60^\circ + i \sin 60^\circ = \frac{1}{2} + \frac{\sqrt{3}}{2} i$. $z^2 = \cos 120^\circ + i \sin 120^\circ = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$. $z^3 = \cos 180^\circ + i \sin 180^\circ = -1$. $z^r = \cos (r\, 60^\circ) + i \sin (r\, 60)$. **3.** $z = \cos 150^\circ + i \sin 150^\circ = -\frac{\sqrt{3}}{2} + \frac{i}{2}$. $z^2 = \cos 300^\circ + i \sin 300^\circ = \frac{1}{2} - \frac{\sqrt{3}}{2} i$. $z^4 = \cos 600^\circ + i \sin 600^\circ = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$. $z^6 = \cos 900^\circ + i \sin 900^\circ = -1$. **4.** $z = \sqrt{2}(\cos 225^\circ + i \sin 225^\circ) = -1 - i$. $z^5 = 4\sqrt{2}(\cos 1125^\circ + i \sin 1125^\circ) = 4(1 + i)$. $z^{10} = 32(\cos 2250^\circ + i \sin 2250^\circ) = 32 i$.

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3. (1) $z^5 = 32 = 32(\cos 2\,k\pi + i \sin 2\,k\pi)$. $z = 2\left(\cos \frac{2\,k\pi}{5} + i \sin \frac{2\,k\pi}{5}\right)$. $z_0 = 2(\cos 0 + i \sin 0) = 2$. $z_1 = 2\left(\cos \frac{2\,\pi}{5} + i \sin \frac{2\,\pi}{5}\right)$. $z_2 = 2\left(\cos \frac{4\,\pi}{5} + i \sin \frac{4\,\pi}{5}\right)$. $z_3 = 2\left(\cos \frac{6\,\pi}{5} + i \sin \frac{6\,\pi}{5}\right)$. $z_4 = 2\left(\cos \frac{8\,\pi}{5} + i \sin \frac{8\,\pi}{5}\right)$. (2) $z^5 = i = \cos\left(\frac{\pi}{2} + 2\,k\pi\right) + i \sin\left(\frac{\pi}{2} + 2\,k\pi\right)$. $z = \cos \frac{90^\circ + 2\,k\,180^\circ}{5} + i \sin \frac{90^\circ + 2\,k\,180^\circ}{5}$. $z_0 = \cos 18^\circ + i \sin 18^\circ$. $z_1 = \cos 90^\circ + i \sin 90^\circ$. $z_2 = \cos 162^\circ + i \sin 162^\circ$. $z_3 = \cos 234^\circ + i \sin 234^\circ$. $z_4 = \cos 306^\circ + i \sin 306^\circ$.

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7. (1) $e^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$; (2) $e^{i\pi} = \cos\pi + i\sin\pi$; (3) $e^{i\frac{2}{3}\pi} = \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi$; (4) $e^{i\frac{\pi}{4}} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$; (5) $e^{-i\pi} = \cos(-\pi) + i\sin(-\pi)$; (6) $e^{-i\frac{\pi}{3}} = \cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$; (7) $e^{-\frac{\pi}{2}} = \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$; (8) $e^{i(\pi+\alpha)} = \cos(\pi+\alpha) + i\sin(\pi+\alpha) = -\cos\alpha - i\sin\alpha$. **8.** (1) $\cos 30^\circ + i\sin 30^\circ = e^{i\frac{\pi}{6}}$; (2) $e^{i\frac{\pi}{4}}$; (3) $e^{i\frac{2}{3}\pi}$; (4) $e^{i\frac{\pi}{3}}$; (5) $e^{i\frac{3\pi}{4}}$; (6) $e^{2-i\frac{\pi}{3}}$. **9.** $\sqrt{e} = 1.648$.

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1. $A = 65^\circ\, 38'\, 35''$, $B = 72^\circ\, 12'\, 22''$, $c = 81^\circ\, 38'\, 45''$. **2.** $A = 56^\circ\, 41'\, 53''$, $B = 57^\circ\, 39'\, 18''$, $b = 50^\circ\, 11'\, 53''$. **3.** $B = 96^\circ\, 20'\, 52''$, $a = 38^\circ\, 20'\, 30''$, $c = 97^\circ\, 57'\, 28''$. **4.** $A = 120^\circ\, 8'\, 14''$, $b = 99^\circ\, 46'\, 45''$, $c = 85^\circ\, 3'\, 7''$. **5.** $B = 110^\circ\, 10'\, 2''$, $a = 42^\circ\, 40'\, 34''$, $b = 165^\circ\, 31'\, 45''$. **6.** $a = 40^\circ\, 54'\, 5''$, $b = 75^\circ\, 39'\, 48''$, $c = 79^\circ\, 12'\, 46''$. **7.** $A = 72^\circ\, 2'\, 36''$, $B = 42^\circ\, 48'\, 17''$, $a = 63^\circ\, 1'\, 2''$. **8.** $A = 34^\circ\, 13'\, 50''$, $a = 32^\circ\, 57'\, 18''$, $c = 75^\circ\, 15'$, or $A_1 = 145^\circ\, 46'\, 10''$, $a_1 = 147^\circ\, 2'\, 42''$, $c_1 = 104^\circ\, 45'$. **9.** $A = 129^\circ\, 14'\, 53''$, $B = 102^\circ\, 10'\, 19''$, $c = 79^\circ\, 50'\, 59''$. **10.** $a = 98^\circ\, 40'\, 57''$, $b = 104^\circ\, 19'\, 26''$, $c = 54^\circ\, 6'\, 46''$. **11.** 46.65 in. **12.** 565.49 sq. in. **14.** (1) Impossible; (2) $b = c = 32^\circ\, 31'\, 30''$, $B = C = 56^\circ\, 49'\, 45''$, or $b_1 = c_1 = 147^\circ\, 28'\, 30''$, $B_1 = C_1 = 123^\circ\, 10'\, 15''$; (3) $C = A = 122^\circ\, 56'\, 10''$, $b = 77^\circ\, 44'\, 30''$. **15.** (1) $A = 121^\circ\, 49'\, 18''$, $b = 60^\circ\, 59'\, 54''$, $c = 46^\circ\, 26'$; (2) $B = 91^\circ\, 24'\, 21''$, $C = 98^\circ\, 7'\, 18''$, $a = 99^\circ\, 54'\, 6''$.

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16. Lat. 40° 6′ 55″ N., Course E. 5° 50′ 17″ S. **17.** Course W. 31° 8′ 17″ S., Distance 3533.6 miles. **18.** Lat. 41° 3′ 56″ N., Long. 53° 58′ 41″ W. **19.** Face angle $BOA = 76^\circ\, 7'\, 32''$, Diedral angles $OA = 74^\circ\, 25'\, 23''$, $OB = 49^\circ\, 56'\, 58''$. **20.** 54° 44′ 7″.

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1. $a = 94^\circ\, 12'\, 28''$, $b = 78^\circ\, 0'\, 32''$, $c = 66^\circ\, 36'\, 26''$. **2.** $A = 36^\circ\, 35'\, 12''$, $B = 72^\circ\, 32'\, 18''$, $C = 83^\circ\, 8'\, 28''$. **3.** $A = 57^\circ\, 38'\, 16''$, $B = 126^\circ\, 57'\, 4''$, $C = 67^\circ\, 9'\, 24''$. **4.** $a = 98^\circ\, 57'\, 28''$, $b = 74^\circ\, 5'\, 46''$, $c = 98^\circ\, 57'\, 28''$. **5.** $a = 30^\circ\, 42'\, 42''$, $b = 47^\circ\, 22'\, 14''$, $C = 79^\circ\, 15'$. **6.** $a = 68^\circ\, 37'$, $B = 132^\circ\, 39'\, 17''$, $C = 52^\circ\, 41'\, 45''$. **7.** $A = 127^\circ\, 14'\, 9''$, $B = 33^\circ\, 37'\, 19''$, $c = 41^\circ\, 20'\, 40''$.

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8. $A = 33^\circ\, 10'\, 25''$, $b = 73^\circ\, 47'\, 30''$, $c = 40^\circ\, 21'\, 48''$. **9.** $A = 44^\circ\, 17'\, 56''$, $B = 48^\circ\, 51'\, 56''$, $c = 159^\circ\, 47'\, 48''$. **10.** No solution. **11.** $A = 52^\circ\, 18'\, 30''$, $a = 40^\circ\, 46'\, 38''$, $C = 39^\circ\, 39'\, 56''$. **12.** $B = 92^\circ\, 38'\, 19''$, $b = 98^\circ\, 55'\, 4''$, $C = 63^\circ\, 35'\, 20''$ or $B = 219^\circ\, 24'\, 44''$, $b = 201^\circ\, 44'$, $C = 116^\circ\, 24'\, 40''$. **13.** $A = 69^\circ\, 48'\, 14''$, $a = 46^\circ\, 45'\, 42''$, $C = 69^\circ\, 54'\, 24''$ or $A = 18^\circ\, 34'\, 30''$, $a = 14^\circ\, 18'\, 52''$, $C = 110^\circ\, 5'\, 36''$. **14.** $B = 59^\circ\, 43'\, 7''$, $C = 218^\circ\, 52'$, $c = 222^\circ\, 8'$, or $B = 120^\circ\, 16'\, 53''$, $C = 68^\circ\, 39'\, 56''$, $c = 96^\circ\, 5'\, 52''$. **15.** No solution. **16.** $B = 54^\circ\, 25'\, 40''$, $b = 116^\circ\, 42'\, 18''$, $c = 27^\circ\, 16'\, 50''$, or $B = 341^\circ\, 56'\, 26''$, $b = 340^\circ\, 5'\, 42''$, $c = 152^\circ\, 43'\, 10''$. **17.** No solution. **18.** $a = 59^\circ\, 55'$, $B = 120^\circ\, 17'\, 20''$, $b = 84^\circ\, 50'\, 28''$, or $a = 120^\circ\, 5'$, $B = 158^\circ\, 42'\, 52''$, $b = 155^\circ\, 14'\, 50''$. **19.** $A = 46^\circ\, 13'\, 46''$, $b = 102^\circ\, 30'\, 16''$, $c = 98^\circ\, 15'\, 8''$. **20.** $A = 117^\circ\, 10'\, 32''$, $B = 145^\circ\, 36'\, 8''$, $c = 44^\circ\, 16'\, 55''$. **21.** $a = 69^\circ\, 17'\, 45''$. **22.** $c = 67^\circ\, 17'\, 52''$. **23.** 74.35 in. **24.** 52.16 in. **25.** 669.2 sq. in. **26.** 162.1 sq. in. **27.** 104.08 sq. in. **28.** 148.31 sq. in. **29.** 7,437,800.0 sq. mi. **30.** 71,515,000 sq. mi., or 27,558,000 sq. mi.

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1. Distance = 2706 knots, bearing of Boston from Liverpool N. 75° 19′ 42″ W., of Liverpool from Boston N. 51° 18′ 4″ E. **2.** 2565.9 miles. **3.** Distance = 3010.26 knots, bearing of New York from Greenwich N. 71° 34′ 22″ W., Greenwich from New York N. 51° 12′ 50″ E. **4.** (1) Lat. 39° 15′ 45″ N., Long.

57° 23′ 21″ W.; (2) Lat. 33° 23′ 38″ N., Long. 34° 37′ 8″ W. **5**. 7423 miles, bearing of Calcutta from San Francisco N. 30° 54′ 53″ W., of San Francisco from Calcutta N. 26° 4′ 31″ E.

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1. Observation in forenoon at 53 min. 44 sec. after 8 o'clock. **2**. 46 min. 53 sec. after 1 o'clock, P.M. **3**. Sunrise at 40 min. 14 sec. after 4 o'clock. **4**. Sunrise at 34 min. 1 sec. after 5 o'clock